Improvements in the Waveform Relaxation Method Applied to Transmission Lines

F. C. M. Lau and E. M. Deeley

Abstract—**The waveform relaxation method has been shown by Chang to have substantial advantages in computation terms when applied to transmission line problems. It is shown in the present paper that the algorithm can break down if d.c. components are present in the signal, and an improved algorithm is presented. Computational results are presented and compared with the HSPICE solution.**

I. Introduction

The method of characteristics [1], [2] has been used for the analysis of transmission lines for many years while waveform relaxation methods [3]–[5] emerged a decade ago to provide more efficient algorithms for solving large-scale electrical circuits. Recently, the method of characteristics has been generalized by Chang [6] for waveform relaxation analysis so that time-domain simulations of lumped-parameter networks interconnected with coupled transmission lines can be carried out more efficiently. The purpose of this paper is to show that in implementing this method, problems concerning the dc components of the solutions can arise, leading to a complete breakdown of the iterative process. The reasons for this are examined and a modified version of the iterative algorithm is given, together with a comparison of results using both algorithms.

II. The Problem with DC Components

It is the purpose of the present paper to show that the solution waveforms given by Chang [6] are applicable to all frequency components except dc, and that this has an important effect on the solution process. Suppose that there is only one RCL transmission line, as shown in Fig. 1, then the iterative solution by Chang [6] will be simplified to

$$V_A^{(k)}(s) = (1 + H + H^2 + \cdots + H^{k-2})U_A + H^{k-1}V_A^{(1)}(s) \tag{1a}$$

$$= \frac{1 - H^{k-1}}{1 - H}U_A + H^{k-1}V_A^{(1)}(s) \text{ if } |H| \neq 1 \tag{1b}$$

$$V_B^{(k)}(s) = (1 + H + H^2 + \cdots + H^{k-2})U_B + H^{k-1}V_B^{(1)}(s) \tag{2a}$$

$$= \frac{1 - H^{k-1}}{1 - H}U_B + H^{k-1}V_B^{(1)}(s) \text{ if } |H| \neq 1 \tag{2b}$$

where

$$U_A = \left(\frac{1}{2}\right)\{(1 - \rho_A)[1 + \rho_B \exp(-2\theta)]E_A + (1 + \rho_A)(1 - \rho_B)\exp(-\theta)]E_B\} \tag{3a}$$

$$U_B = \left(\frac{1}{2}\right)\{(1 - \rho_A)(1 + \rho_B)\exp(-\theta)E_A + [1 + \rho_A \exp(-2\theta)](1 - \rho_B)E_B\} \tag{3b}$$

$$H = \rho_A\rho_B \exp(-2\theta) \tag{3c}$$

$$\rho_A = (Z_A - Z_0)/(Z_A + Z_0) \tag{3d}$$

$$\rho_B = (Z_B - Z_0)/(Z_B + Z_0). \tag{3e}$$

Manuscript received December 9, 1992; revised September 1, 1994. This work was supported by General Electric Company (Schenectady, NY) and the University of London.

F. C. M. Lau is with the Department of Electronic Engineering, Hong Kong Polytechnic University, Kowloon, Hong Kong.

E. M. Deeley is with the Department of Electronic and Electrical Engineering, King's College London, University of London, London WC2R 2LS, U.K.

IEEE Log No. 9410348.

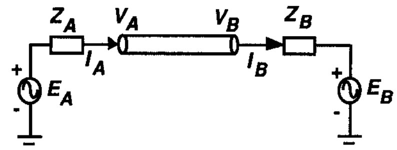


Fig. 1. Single RCL transmission line terminated in Thevenin's equivalent circuit.

To evaluate the dc components of the solutions, $s = j\omega = 0$ is substituted. Since Z_0 has been approximated by m symmetrical T-networks terminated in R_S (Pade's approximation) and its dc resistance is $(2m + 1)R_S$, therefore

$$Z_0(0) = (2m + 1)R_s \tag{4a}$$

$$\theta(0) = 0 \tag{4b}$$

and

$$\rho_A = \frac{R_A - Z_0}{R_A + Z_0} \neq 1 \text{ and } |\rho_A| < 1 \tag{5a}$$

$$\rho_B = \frac{R_B - Z_0}{R_B + Z_0} \neq 1 \text{ and } |\rho_B| < 1 \tag{5b}$$

$$|H| = |\rho_A\rho_B| < 1. \tag{5c}$$

The equations are applied to (1b) and (2b) to give

$$V_A^{(\infty)}(0) = V_B^{(\infty)}(0) = \frac{R_B E_{A\mathrm{dc}} + R_A E_{B\mathrm{dc}}}{R_A + R_B} \tag{6}$$

where $E_{A\mathrm{dc}}$ and $E_{B\mathrm{dc}}$ are the dc components of the Thevenin equivalent voltage sources at the ends. The dc components at the transmission-line ends have converged to the same value which indicates that the iterative algorithm in [6] does not take into account the resistive loss of the transmission line as far as the dc components are concerned. The solution is also independent of the number of T-networks used to approximate Z_0. The main problem lies on the fact that the T-network used to approximate Z_0 has a finite dc resistance while theoretically, it should be infinite.

The correct dc components of the system can be found by replacing the transmission line by an equivalent resistor of value R_0 = Resistivity $*$ length and solving the resulting system, which gives

$$V_{A\mathrm{dc}} = \frac{(R_B + R_0)E_{A\mathrm{dc}} + R_A E_{B\mathrm{dc}}}{R_A + R_B + R_0} \tag{7a}$$

$$V_{B\mathrm{dc}} = \frac{R_B E_{A\mathrm{dc}} + (R_A + R_0)E_{B\mathrm{dc}}}{R_A + R_B + R_0}. \tag{7b}$$

Comparing (6) and (7), the dc component errors induced in the iteration algorithm are small only if $R_B \gg R_0$ and $R_A \gg R_0$, or if the dc components of the Thevenin voltage sources are negligible.

Since this problem appears in the waveform relaxation analysis of a single RCL transmission line, it also occurs in the iterative algorithm proposed by Chang. A modified waveform relaxation algorithm will be proposed in the next section which takes into account the dc components.

0018–9480/95$04.00 © 1995 IEEE

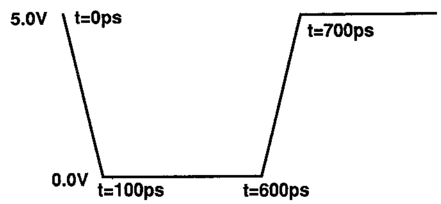


Fig. 2. Waveform for voltage source $E_{1a}(t)$.

III. Modified Waveform Relaxation Algorithm

The algorithm is similar to that used by Chang [6] except that two modifications are made to Step 2 and 4 which are described below.

Step 2a: Compute $\{\mathbf{w}_b^{k-1/2}(t)\}$ from $\{\mathbf{v}_a^k(t)\}$ and $\{\mathbf{w}_a^{k-1}(t)\}$ by the FFT and the inverse FFT

$$\mathbf{w}_b^{k-1/2} = F^{-1}\{\mathbf{H} * F[2\mathbf{X}^{-1}\mathbf{v}_a^k - \mathbf{w}_a^{k-1}]\}.$$

Step 2b: Shift the waveforms $\{\mathbf{w}_b^{k-1/2}(t)\}$ by $\{\mathbf{w}_{b0} - \mathbf{w}_b^{k-1/2}(t=0)\}$ and store the result in the array

$$\mathbf{w}_b^k = \mathbf{w}_b^{k-1/2} + [\mathbf{w}_{b0} - \mathbf{w}_b^{k-1/2}(t=0)].$$

Step 4a: Compute $\{\mathbf{w}_a^{k-1/2}(t)\}$ from $\{\mathbf{v}_b^k(t)\}$ and $\{\mathbf{w}_b^k(t)\}$ by the FFT and the inverse FFT

$$\mathbf{w}_a^{k-1/2} = F^{-1}\{\mathbf{H} * F[2\mathbf{X}^{-1}\mathbf{v}_b^k - \mathbf{w}_b^k]\}.$$

Step 4b: Shift the waveforms $\{\mathbf{w}_a^{k-1/2}(t)\}$ by $\{\mathbf{w}_{a0} - \mathbf{w}_a^{k-1/2}(t=0)\}$ and store the result in the array

$$\mathbf{w}_a^k = \mathbf{w}_a^{k-1/2} + [\mathbf{w}_{a0} - \mathbf{w}_a^{k-1/2}(t=0)].$$

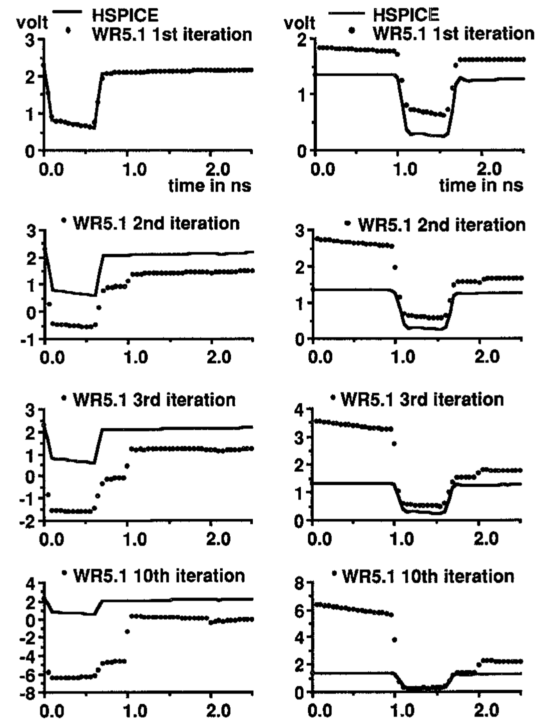
where $\mathbf{w}_{a0}$ and $\mathbf{w}_{b0}$ are the values of the voltage generators evaluated by the initial condition analysis.

Proof: It has been realized that the waveform relaxation algorithm proposed by Chang [6] has difficulty only in deriving the dc components of the terminal voltage waveforms, which are given in (7a) and (7b). Prior to performing the iterations, however, similar equations have actually been applied to evaluate the initial conditions of the terminal voltages

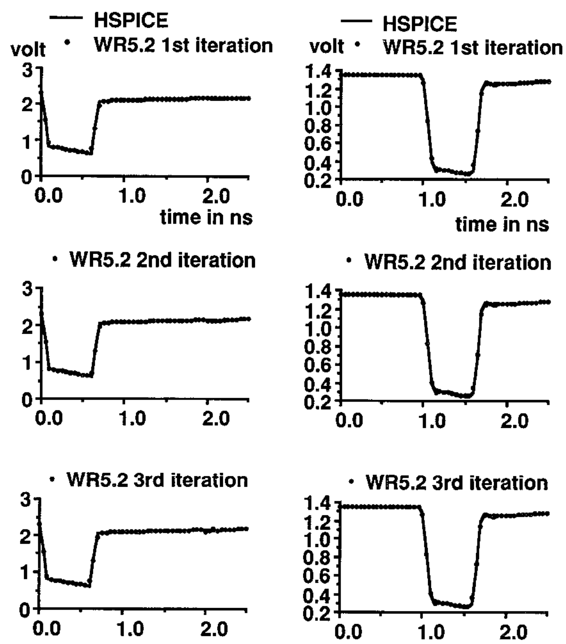
$$V_{A0} = \frac{(R_B + R_0)E_{A0} + R_A E_{B0}}{R_A + R_B + R_0} \quad (8a)$$

$$V_{B0} = \frac{R_B E_{A0} + (R_A + R_0)E_{B0}}{R_A + R_B + R_0}. \quad (8b)$$

The initial values of the voltage generators $\mathbf{w}_{a0}$ and $\mathbf{w}_{b0}$ are then derived from the above initial conditions. Hence, if the iterative procedures can take care of the dc components, the initial values of the voltage generators, evaluated by FFT and IFFT, $\mathbf{w}_a^{k-1/2}$ and $\mathbf{w}_b^{k-1/2}$ must satisfy these initial conditions. But due to the dc problem, the voltage generator waveforms do not have the correct dc value. Since the correct initial values of the generators are known, the dc level of the generators can be shifted appropriately such that the initial conditions, and hence the dc level, are satisfied. The new algorithm is found to generate correct dc components for the solutions. □



(a)



(b)

Fig. 3 (a) Iterative waveform simulation of near-end terminal voltage waveforms (left sequence of waveforms) and far-end terminal voltage waveforms (right sequence of waveforms) of the active outer transmission line using algorithm **WR5.1**. (b) Iterative waveform simulation of near-end terminal voltage waveforms (left sequence of waveforms) and far-end terminal voltage waveforms (right sequence of waveforms) of the active outer transmission line using algorithm **WR5.2**.

IV. Some Simulation Results

Simulations have been run on circuits using the waveform relaxation algorithms proposed by Chang and by the present method, **WR5.1** and **WR5.2** respectively, one of which is given below.

A triconductor stripline system has been analysed with the parameters given in Appendix A using the input waveform $E_{1a}(t)$ depicted in Fig. 2. The characteristic impedances are each approximated by three sections of T-networks in the waveform relaxation algorithms. A window enlargement factor of 4 and 1024 samples are used in the FFT simulation of the exponential wave propagation function of the lossy lines.

The system has also been simulated by the conventional circuit simulator HSPICE where the coupled lossy transmission lines are represented by 100 symmetrical blocks (Appendix B) connected in tandem. Extremely small time steps of 0.2 ps have been used in HSPICE to eliminate false ringing effects.

The waveforms at the near and far ends of the active outer transmission line generated by HSPICE are compared with those produced by **WR5.1** and **WR5.2**, as depicted in Figs. 3(a) and (b), respectively. It is clearly shown in Fig. 3(a) that the sequences of waveforms generated by **WR5.1** are diverging from the correct solutions. This is due to the incapability of the algorithm to handle situations when dc components exist. The iteration is stopped after the 10-iterations limit is exceeded.

On the other hand, **WR5.2**, generates sequences of waveforms which converge to the same solutions predicted by HSPICE, as in Fig. 3(b). It takes **WR5.2** only two iterations to arrive at the correct waveforms and the third iteration is performed to confirm convergence.

HSPICE has taken 5254 seconds to generate the waveforms while **WR5.2** spends just only 9.92 seconds to perform the three iterations, which is about 500 times faster. This shows the great effectiveness and efficiency possessed by the waveform relaxation algorithm.

V. Conclusion

The method of characteristics has been generalized for waveform relaxation analysis by Chang, enabling the time-domain simulations of lumped-parameter networks interconnected with coupled transmission lines with conductor loss to be carried out more efficiently. In this paper the theory has been examined thoroughly, which reveals that the solution process would break up should dc components exist in the solutions. A modified version of the iterative algorithm has been suggested and simulated results using linear terminations confirm the effectiveness of the modified algorithm compared with the original one. When the terminations are nonlinear, the dc levels at the ends cannot be derived so easily and in a separate article [7] the problem will be investigated and solved.

Appendix A
Triconductor Stripline Parameters

The triconductor stripline system has been analysed with the following parameters:

$$\mathbf{L} = \begin{bmatrix} 3 & 1 & 0.5 \\ 1 & 4 & 1 \\ 0.5 & 1 & 3 \end{bmatrix} \text{nH/cm}$$

$$\mathbf{C} = \left(\frac{1}{24}\right)\begin{bmatrix} 44 & -10 & -4 \\ -10 & 35 & -10 \\ -4 & -10 & 44 \end{bmatrix} \text{pF/cm}$$

$$\tilde{\mathbf{L}} = \text{diag}(5/3, 5, 2.5)\ \text{nH/cm}$$

$$\mathbf{R} = \text{diag}(2.5, 2.0, 1.5)\ \Omega/\text{cm}$$

length $l = 14.4$ cm

$$Z_A = \text{diag}(100, 200, 100)\ \Omega$$

$$Z_B = \text{diag}(50, 75, 50)\ \Omega$$

$$E_{2a}(t) = E_{3a}(t) = E_{1b}(t) = E_{2b}(t) = E_{3b}(t) = 0.0 \text{ for all } t.$$

Appendix B
Symmetrical RCL Block Used in HSPICE

Fig. 4 shows the symmetrical block used in modeling coupled lossy transmission lines in HSPICE. The values of the lumped elements in the block is given by

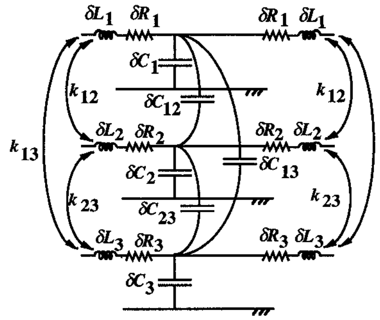


Fig. 4. Symmetrical block used in modeling coupled lossy transmission lines in HSPICE.

$$\delta L_i = \frac{L_{ii} * l}{2 * \text{no. of symmetrical blocks}} \quad i = 1, 2, 3$$

$$L_{ii} = i\text{th diagonal element of } \mathbf{L}$$

$$\delta C_i = \frac{\left(\sum_{j=1}^{3} C_{ij}\right) * l}{\text{no. of symmetrical blocks}} \quad i = 1, 2, 3$$

$$C_{ij} = i, j\text{th element of } \mathbf{C}$$

$$\delta C_{ij} = \frac{|C_{ij}| * l}{\text{no. of symmetrical blocks}} \quad 1 \le i \le j \le 3$$

$$C_{ij} = i, j\text{th element of } \mathbf{C}$$

$$\delta R_i = \frac{R_{ii} * l}{2 * \text{no. of symmetrical blocks}} \quad i = 1, 2, 3$$

$$R_{ii} = i\text{th diagonal element of } \mathbf{R}$$

and the mutual inductance coefficients are

$$k_{ij} = \frac{L_{ij}}{\sqrt{L_i L_j}} \quad 1 \le i \le j \le 3 \quad L_{ij} = i, j\text{th element of } \mathbf{L}.$$

References

[1] F. H. Branin, Jr., "Transient analysis of lossless transmission line," *Proc. IEEE*, vol. 55, pp. 2012–2013, Nov. 1967.

[2] F. Y. Chang, "Transient analysis of lossless coupled transmission lines in a homogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 616–626, Sept. 1970.

[3] E. Lelarasmee, A. E. Ruehli, and A. L. Sangiovanni-Vincentelli, "The waveform relaxation method for time-domain analysis of large scale integrated circuits," *IEEE Trans. Computer-Aided Design*, vol CAD-1, no. 3, pp. 131–145, July 1982.

[4] D. J. Erdmann and D. J. Rose, "A Newton waveform relaxation algorithm for circuit simulation," in *1989 IEEE Int. Conf. Computer-Aided Design, Dig. Tech. Papers*.

[5] Y. H. Jun, C. W. Lee, K. J. Lee, and S. B. Park, "Timing simulator by waveform relaxation considering the feedback effect," *IEE Proc.*, pt. G, vol. 136, no. 1, pp. 38–42, Feb. 1989.

[6] F. Y. Chang, "The generalized method of characteristics for waveform relaxation analysis of lossy coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 12, pp. 2028–2038, Dec. 1989

[7] F. C. M. Lau, "Improvements in the waveform relaxation method applied to transmission lines with nonlinear terminations," in preparation.